Abstract
This paper describes Scilab scripts that are used to identify vented-box parameters from impedance measurements. These scripts use non-linear least squares optimization to estimate the enclosure losses (port, leakage and absorption losses), the system tuning ratio and the system compliance ratio. Additionally, these scripts calculate the free-field frequency response based on measurement of the acoustical pressure within the enclosure.

Nomenclature
\( \alpha \) System compliance ratio = \( \frac{V_{as}}{V_{ab}} \)
\( \beta \) Apparent volume to net enclosure volume ratio = \( \frac{V_{ab}}{V_{b}} \)
\( \mathcal{G}(S) \) System response function
\( \mathcal{P}_b \) Pressure function inside the enclosure
\( \mathcal{P}_e \) Free-field pressure function
\( \mathcal{P}_g \) Acoustic pressure generator = \( BlU_g/S_dR_e \)
\( \mathcal{P}_b \) Enclosure volume velocity
\( \mathcal{P}_d \) Loudspeaker volume velocity
\( \mathcal{P}_l \) Leakage volume velocity
\( \mathcal{P}_p \) Vent volume velocity
\( \mathcal{U}_g \) Electric source output voltage
\( \mathcal{Z}(S) \) Impedance function of the loudspeaker mounted in enclosure
\( \rho \) Air density
\( B \) Magnetic flux density in driver air gap
\( c \) Speed of sound
\( C_{ab} \) Acoustic compliance of air in enclosure
\( C_{as} \) Acoustic compliance of driver suspension
\( C_{eo} \) Electrical capacitance due to driver mass = \( M_{ao}^{-1}S_d^2/(Bl)^2 \)
\( C_{ep} \) Electrical capacitance due to vent mass = \( M_{ap}^{-1}S_d^2/(Bl)^2 \)
\( f \) Current frequency
\( f_p \) Resonance frequency of vented enclosure = \( 1/2\pi\sqrt{M_{ap}^{-1}C_{ab}} \)
\( f_s \) Resonance frequency of driver = \( 1/2\pi\sqrt{M_{as}^{-1}C_{as}} \)
\( f_{so} \) Resonance frequency of driver mounted in enclosure = \( 1/2\pi\sqrt{M_{as}^{-1}C_{as}} \)
\( h \) System tuning ratio = \( f_p/f_{so} \)
\( l \) Effective length of voice coil conductor
\( L_{cb} \) Electrical inductance due to enclosure compliance = \( C_{ab}^{-1}S_d^2/(Bl)^2 \)
\( L_{es} \) Electrical inductance due to driver suspension compliance = \( C_{as}^{-1}S_d^2/(Bl)^2 \)
\( M_{ao} \) Acoustic mass of driver diaphragm including air load when mounted in enclosure
\( M_{ap} \) Acoustic mass of port including air load
\( M_{as} \) Acoustic mass of driver diaphragm including air load
\( q \) Driver acoustic mass ratio = \( M_{ao}^{-1}/M_{as}^{-1} \)
\( Q_a \) Enclosure \( Q \) at \( f_p \) resulting from absorption losses = \( 1/2\pi f_p C_{ab} R_{ab} \)
\( Q_l \) Enclosure \( Q \) at \( f_p \) resulting from leakage losses = \( 2\pi f_p C_{ab} R_{ld} \)
\( Q_p \) Enclosure \( Q \) at \( f_p \) resulting from vent frictional losses = \( 1/2\pi f_p C_{ab} R_{ap} \)
\( Q_{eo} \) Electric driver losses at \( f_{so} \) when mounted in enclosure = \( 1/2\pi f_{so} C_{as} R_{as} \)
\( Q_{es} \) Electric driver losses at \( f_s \)
\( Q_{ms} \) Mechanical driver losses at \( f_{so} \) when mounted in enclosure = \( 1/2\pi f_{so} C_{as} R_{as} \)
\( Q_{ms} \) Mechanical driver losses at \( f_s \)
\( Q_{to} \) Total driver \( Q \) at \( f_{so} \) resulting from electric and mechanical driver losses when mounted in enclosure
\( r \) Distance from the loudspeaker
\( R_{ab} \) Acoustic resistance of enclosure losses caused by internal energy absorption
The Thiele and Small parameters of the loudspeaker

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In order to be able to accurately simulate the low-
frequency response of a vented-box loudspeaker sys-
tem, it is necessary to know the enclosure losses, the
Thiele and Small parameters of the driver, the reso-
nant frequency of the vented enclosure and the vol-
ume which represents the acoustic compliance of the
enclosure. Determination of these parameters is not
without problems.

- Three kinds of enclosure losses have to be taken into
account: the leakage losses \( Q_l \), the absorption losses
\( Q_a \) and the vent losses \( Q_v \). Magnitudes of these
losses have been determined by Small in [2]. Typical
values for \( Q_l \) are in range 50-100. Typical value
for \( Q_a \) is 100 for unlined enclosures and between 30-
80 when lining material is placed on the enclosure.
Small consider that leakage losses are the most sig-
ificant giving \( Q_l \) values between 5 and 20. These
recommendations have been taken by WinISD pro
[5] which use \( Q_l \) of 10 by default.

- The Thiele and Small parameters of the loudspeaker
can be taken from the manufacturer. These param-
eters are \( R_{ae}, f_a, Q_{cs}, Q_{ms}, V_{as} \). However, production
tolerance are such that it is preferable to measure
them, and it is imperative to do it in case of old
loudspeakers. The measurement of the loudspeaker
should be such that it is loaded by the same air load
mass as when it is mounted in the vented enclosure.
The best way to achieve that is to mount the loud-
speaker in a baffle such as the one recommended
in the IEC 268 standard. However the size of this
baffle for a 15 inch loudspeaker is very huge and it
is more convenient to measure the driver free-air.
This leads to different air load mass and the rezo-
nance frequency of driver mounted in the enclosure
\( f_{so} \) will differ from the free-air resonance frequency
\( f_s \). As a consequence the values \( Q_{ea}, Q_{mo} \) will also
differ from \( Q_{es}, Q_{ms} \).

Another problem is related to the measure of the
\( V_{as} \) parameter. The easiest and quicker method is
the Delta mass method. However a single measure-
ment with such a technique cannot provide better
accuracy than 5 %.

- The resonant frequency of the vented enclosure \( f_p \)
is computed from port characteristics. Port end-
correction depends on the number of vents, their
shapes and their mounting (flush-mounted or not).
Different empirical formulas are used for calculat-
ing this correction. None of these formulas is very
accurate.

- The volume which represents the acoustic compli-
ance of the enclosure will differ from the net inter-

Introduction

As we can see, parameters needed to simulate the
low frequency response of a vented-box, which are
\( Q_l, Q_a, Q_v, f_{so}, Q_{ea}, Q_{mo}, f_p, \beta \), are not easily deter-

The purpose of this paper is to study how these
parameters can be identified and provide Scilab scripts
to do it.

1 Acoustical equivalent circuit of
a vented-box loudspeaker sys-
tem

Referring to small paper [2] the acoustical analogous
circuit of a vented-box is presented figure [1].

Using overline notation for Laplace function, the
system response function is given by:

\[
\overline{G}(S) = \frac{a_4 S^4 + a_3 S^3}{a_4 S^4 + a_3 S^3 + a_2 S^2 + a_1 S + a_0}
\]  (1)
Figure 1: Acoustical analogous circuit of vented-box loudspeaker system

The coefficients are:

\[
\begin{align*}
  a_0 &= h^3(1 + Q_p^{-1}Q_l^{-1}) \\
  a_1 &= h^3Q_{p0}^{-1}(1 + Q_p^{-1}Q_l^{-1}) + h^2\alpha Q_p^{-1} \quad (2) \\
  &\quad + h^2(Q_p^{-1} + Q_a^{-1} + Q_l^{-1} + Q_p^{-1}Q_a^{-1}Q_l^{-1}) \\
  a_2 &= h^3(1 + Q_p^{-1}Q_l^{-1}) \quad (3) \\
  &\quad + h^2Q_{p0}^{-1}(Q_p^{-1} + Q_a^{-1} + Q_l^{-1} + Q_p^{-1}Q_a^{-1}Q_l^{-1}) \\
  &\quad + h(\alpha(1 + Q_p^{-1}Q_l^{-1}) + 1 + Q_a^{-1}Q_l^{-1}) \\
  a_3 &= h^2(Q_p^{-1} + Q_a^{-1} + Q_l^{-1} + Q_p^{-1}Q_a^{-1}Q_l^{-1}) \quad (4) \\
  a_4 &= h(1 + Q_a^{-1}Q_l^{-1}) \quad (5) \\
  b_3 &= h^2Q_p^{-1}(1 + Q_a^{-1}Q_l^{-1}) \quad (6) \\
  &\quad + h\alpha Q_a^{-1} \\
  b_1 &= h(1 + Q_a^{-1}Q_l^{-1}) \quad (7)
\end{align*}
\]

The system response is a fourth-order high-pass filter function depending only on \(f_{so}, Q_{to}, Q_l, Q_a, Q_p, h, \alpha\).

The effects of enclosure losses on response is shown on figure 2 where the lossless curve is a B-4 aligned vented-box loudspeaker system.

![Figure 2: Effects of Q losses on response of a vented-box loudspeaker system](image)

It can be seen that enclosure losses do not depend directly on \(V_{as}\) but only on \(\alpha\), the system compliance ratio:

\[\alpha = \frac{V_{as}}{\beta V_b}\]  

(8)

From this equation we can conclude that:

- An accurate measurement of \(V_{as}\) is not needed because the true value of \(\beta\) is unknown. In most cases, the \(V_{as}\) value can be taken directly from the manufacturer of the loudspeaker.

- The internal volume \(V_b\) can be adjusted to reach the target value of \(\alpha\) for the true values of \(V_{as}\) and \(\beta\). It is therefore possible to get the exact desired system response even if true values of \(V_{as}\) and \(\beta\) are initially unknown. This can be done by building the enclosure box with an initial higher volume which will be decreased during the optimization process.

2 Electrical equivalent circuit of a vented-box loudspeaker system

The electrical equivalent circuit of the vented-box loudspeaker system is formed by taking the dual of the acoustic circuit of figure 1 and converting each element to its electrical equivalent. We get the circuit of figure 0.

From this figure we can calculate the electrical impedance function:

\[Z(S) = R_e \frac{b_3S^3 + b_2S^2 + b_1S + b_0}{a_4S^3 + a_3S^2 + a_2S + a_1S + a_0}\]  

(9)

The coefficients are:

\[
\begin{align*}
  a_0 &= h^3(1 + Q_p^{-1}Q_l^{-1}) \quad (10) \\
  a_1 &= h^3Q_{p0}^{-1}(1 + Q_p^{-1}Q_l^{-1}) + h^2\alpha Q_p^{-1} \quad (11) \\
  &\quad + h^2(Q_p^{-1} + Q_a^{-1} + Q_l^{-1} + Q_p^{-1}Q_a^{-1}Q_l^{-1}) \\
  a_2 &= h^3(1 + Q_p^{-1}Q_l^{-1}) \quad (12) \\
  &\quad + h^2Q_{p0}^{-1}(Q_p^{-1} + Q_a^{-1} + Q_l^{-1} + Q_p^{-1}Q_a^{-1}Q_l^{-1}) \\
  &\quad + h(\alpha(1 + Q_p^{-1}Q_l^{-1}) + 1 + Q_a^{-1}Q_l^{-1}) \\
  a_3 &= h^2(Q_p^{-1} + Q_a^{-1} + Q_l^{-1} + Q_p^{-1}Q_a^{-1}Q_l^{-1})
\end{align*}
\]
\[ a_4 = h(1 + Q_a^{-1}Q_l^{-1}) \]  
\[ b_0 = h^3(1 + Q_p^{-1}Q_l^{-1}) \]  
\[ b_1 = h^3Q_{eo}^{-1}(1 + Q_p^{-1}Q_l^{-1}) + h^2aQ_p^{-1} \]  
\[ + h^2(1 + Q_p^{-1}Q_l^{-1} + Q_l^{-1} + Q_p^{-1}Q_a^{-1}Q_l^{-1}) \]  
\[ b_2 = h^2Q_{eo}^{-1}(Q_p^{-1} + Q_a^{-1} + Q_l^{-1} + Q_p^{-1}Q_a^{-1}Q_l^{-1}) \]  
\[ + h(a(1 + Q_p^{-1}Q_a^{-1} + 1 + Q_a^{-1}Q_l^{-1}) \]  
\[ b_3 = h^2(Q_p^{-1} + Q_a^{-1} + Q_l^{-1} + Q_p^{-1}Q_a^{-1}Q_l^{-1}) \]  
\[ + hQ_{eo}^{-1}(1 + Q_a^{-1}Q_l^{-1}) + aQ_a^{-1} \]  
\[ b_4 = h(1 + Q_a^{-1}Q_l^{-1}) \]

The figure 4 depicts residuals, i.e. differences between the measurements values and the fitted values provided by the model.

![Impedance curves and residual adjustments](image)

As expected, residuals are zero because we used the same model to simulate the measurements and identify the parameters. However, the solution does not converge to the initial settings. Apart from \( f_{so} \) and \( Q_{eo} \), all other parameters differ.

An observability check can be carried out with the eigenvalues of the normal equation matrix (see annexe A). The smallest eigenvalue is 2.2 \( 10^{-12} \) which indicates that this matrix is singular and that all parameters cannot be identified. The eigenvector coordinates associated with this eigenvalue are :

\[
\begin{array}{cccc}
 f_{so} & Q_{eo} & Q_{mo} & Q_l \\
 2.0 & 0.3 & 7.78 & 18.10 \\
 Q_a & Q_p & h & \alpha \\
 15.60 & 10.26 & 1.4998 & 2.0001 \\
\end{array}
\]

The significant coordinates relate on the parameters \( Q_{mo}, Q_l, Q_a, Q_p \), and to a lesser extent on \( h, \alpha \). The coordinates associated with \( f_{so} \) and \( Q_{eo} \) are almost zero. We can conclude that the lack of observability does not concern \( f_{so} \) and \( Q_{eo} \) but \( Q_{mo}, Q_l, Q_a, Q_p \). This is consistent with what we get from estimated values. Estimations give good values of \( f_{so} \) and \( Q_{eo} \), close values of \( h, \alpha \) and bad values of \( Q_{mo}, Q_l, Q_a, Q_p \).
Signs of the eigenvector coordinates provide the way the estimated parameters vary. It can be seen that a decrease of $Q_{mo}$ will result in a decrease of $Q_t$ and an increase of $Q_a$ and $Q_p$.

The reason for this can be clearly understood from the electric circuit of figure [3]. The resistance $R_{es}$ which define $Q_{mo}$ is in parallel with the resistances $R_{el}, R_{eb}, R_{ep}$ which respectively define $Q_t, Q_a, Q_p$. One understand that an increase of $R_{es}$ can be compensated by a decrease of $R_{el}, R_{eb}, R_{ep}$ to finally get the same value of the impedance. The definitions of $Q$ losses factors are consistent with the variations of $Q_{mo}, Q_t, Q_a, Q_p$ previously observed.

Closer examination of the coefficients of the impedance transfer function shows that there is actually one degree of freedom within $Q_{mo}, Q_t, Q_a, Q_p, h, \alpha$. More details can be found in annexe B of reference [10].

We can conclude therefore that it is not possible to identify simultaneously the four parameters $Q_{mo}, Q_t, Q_a, Q_p$ from impedance measurements.

4 Free-field frequency response measurement

The free-field low frequency response measurement of a loudspeaker system is a difficult task. It cannot be performed in a semi-reverberant room because of the modal response of this room. It cannot also be performed in a standard anechoic chamber because most of them have a frequency cutoff greater than 60 hz. There are mainly two technics to measure the free-field low frequency response. The first one, proposed by D.B. Keele in [3], is based on the measurement taken in the near-field outside the enclosure. The second one, proposed by Small in [4], consists to calculate the free-field frequency response from the measurement of the acoustic pressure within the system enclosure.

The first method is more complicated for a vented-box than for a closed box because it needs to sum the near-field measurements of the loudspeaker diaphragm and vent. The summation implies to be vectorial which means that both magnitude and phase have to be considered. The level of the diaphragm and the vent must be adjusted before the response is summed. The level of the diaphragm and the vent. The summation implies to be vectorial which means that both magnitude and phase have to be considered. The level of the diaphragm and the vent. The summation implies to be vectorial which means that both magnitude and phase have to be considered. The level of the diaphragm and the vent. The summation implies to be vectorial which means that both magnitude and phase have to be considered. The level of the diaphragm and the vent. The summation implies to be vectorial which means that both magnitude and phase have to be considered. The level of the diaphragm and the vent. The summation implies to be vectorial which means that both magnitude and phase have to be considered. The level of the diaphragm and the vent. The summation implies to be vectorial which means that both magnitude and phase have to be considered. The level of the diaphragm and the vent. The summation implies to be vectorial which means that both magnitude and phase have to be considered. The level of the diaphragm and the vent. The summation implies to be vectorial which means that both magnitude and phase have to be considered. The level of the diaphragm and the vent. The summation implies to be vectorial which means that both magnitude and phase have to be considered. The level of the diaphragm and the vent. The summation implies to be vectorial which means that both magnitude and phase have to be considered. The level of the diaphragm and the vent. The summation implies to be vectorial which means that both magnitude and phase have to be considered. The level of the diaphragm and the vent.

The second method requires only a measurement of the pressure within the enclosure regardless of the number of radiating surfaces. However the vent resonant frequency and the absorption losses need to be known to accurately compute the free-field frequency response. This is the method we use in this paper.

Let us denote $P_b$ the measurement of the pressure inside the enclosure. From analysis of the acoustical equivalent circuit of figure [1], the relation between internal pressure and internal volume velocity is:

$$P_b = (R_{ab} + \frac{1}{j2\pi f C_{ab}})\bar{\eta}_b$$

The external pressure outside the enclosure must be computed with the sum of all radiating surfaces (diaphragm, vent and leakage). As we have:

$$\bar{\eta}_b = \bar{\eta}_d + \bar{\eta}_l + \bar{\eta}_p$$

the external pressure can be simply computed with the internal volume velocity, which makes this method very simple. The external pressure, at a distance $r$ from the enclosure, is therefore given by:

$$\bar{P}_e = \frac{\rho f^2}{r} \bar{\eta}_b$$

From definitions of $C_{ab}$ and $R_{ab}$ we get:

$$C_{ab} = \frac{V_b}{\rho c}$$
$$\omega R_{ab} C_{ab} = \int \frac{f}{f_{so}} (hQ_a)^{-1}$$

Combining these equations, we obtain:

$$\bar{P}_e = \frac{\beta V_b 2\pi f_s^2}{r} \frac{S^2}{1 + S(hQ_a)^{-1}} \bar{P}_b$$

This method is valid as long as the pressure inside the enclosure is uniform and the development of standing wave within the enclosure make it useless. Measurements show that the pressure inside the enclosure becomes noticeably non uniform even below the first standing wave. To improve the validity range of this method, the microphone should be placed near the geometrical center of the enclosure.

5 Description of Scilab scripts

Scilab is an open source software for numerical computation. Scilab can be downloaded from [11] and is available for Window, Linux or Mac. Scilab can be used interactively, by typing commands in the console window. Scilab provides also a powerful editor, Scinotes, to edit scripts. Names of script have extension .sce or .sci. The files having extension .sci contains Scilab functions. Executing them loads the functions into the Scilab environment. The files having extension .sce contains executables.

Scilab scripts describe in this paper are <SciAudioBox.sci>, <Measure Vented-Box 1.sce>, <Measure Vented-Box 2.sce> and <Simulate Vented Box.sce>.
The `<SciAudioBox.sci>` is the library which contains the main functions used by others scripts. It must be run once before execution of one of the other scripts. `<Measure Vented-Box 1.sce>` and `<Measure Vented-Box 2.sce>` scripts read impedance measurement from a file to identify vented-box parameters. Impedance measurements must be saved in an ASCII text file. Non-numeric lines are ignored. Data lines must begin with the frequency in Hz, then the impedance magnitude in Ohm and finally the phase in degrees.

`<Simulate Vented Box.sce>` script reads pressure measurement from a file to compute the free-field frequency response. Format of this file is identical to that of the impedance, except that magnitude is in dB.

Softwares like ARTA [6] of REW [7] can be used to measure impedance and acoustic pressure. The exported file formats by these softwares are compatible with the one used by the Scilab scripts.

5.1 Identification of vented-box parameters with a known loudspeaker

In section 3, we have seen that it is not possible to identify simultaneously $Q_{mo}, Q_l, Q_a, Q_p$ from impedance measurements. As a consequence the value of $Q_{mo}$ must be obtained through other means. Unfortunately, the measurement of the loudspeaker gives $Q_{ms}$ but not $Q_{mo}$. Let us denote $q$ the acoustic mass ratio. From definitions of $f_{so}, Q_{eo}$, $Q_{mo}$ and $f_s, Q_{es}, Q_{ms}$ we get:

$$f_{so} = f_s \sqrt{q}, \quad Q_{eo} = \frac{Q_{es}}{\sqrt{q}}, \quad Q_{mo} = \frac{Q_{ms}}{\sqrt{q}}$$

As $f_{so}$ and $Q_{eo}$ are well observed from impedance measurements, it is obvious that knowing $f_s$ and $Q_{es}$ leads to well estimate $q$. It is therefore clear that identifying $q$ instead of $f_{so}, Q_{eo}, Q_{mo}$ will get rid of the lack of observability and make all parameters fully estimated.

This is the purpose of the script `<Measure Vented-Box 1.sce>`.

The figure 5 shows the input parameters of this script.

The user has to enter the following parameters:

- the directory of the impedance measurement file,
- the name of the impedance measurement file,
- the loudspeaker parameters $R_e, f_s, Q_{es}, Q_{ms}$,
- the net volume of enclosure $V_b$ and the volume of air having same compliance as loudspeaker suspension $V_{as}$,
- the initial guess of $q, Q_l, Q_a, Q_p, h, \alpha$.

Figure 5: Input data of Measure Vented-Box 1 script
Running this script with the simulated test case of section 3 leads to perfectly identify parameters used for the simulation.

To quantify the level of observability of identified parameters one can compute the value $p_{\mu_{\text{min}}}$ where $p$ is the number of identified parameters and $\mu_{\text{min}}$ the smallest eigenvalue of the least squares matrix (see annee A). The higher the value, the higher the parameters are identified. The use of only the magnitude of the impedance, leads to a value of 0.02 while the use of both magnitude and phase leads to 0.12. It is therefore recommended to use both magnitude and phase.

5.2 Identification of vented-box parameters with an unknown loudspeaker

When Thiele and Small parameters of the loudspeaker are unknown and cannot be measured, it is always possible to identify the vented-box parameters with a given value of $Q_{\text{mo}}$. This is the purpose of the script <Measure Vented-Box 2.sce>. Input parameters of this script are not very different from the previous one. Instead entering T/S parameters of the loudspeaker, the user has just to enter the known value of $Q_{\text{mo}}$. The accuracy of the results is of course directly related to the accuracy of the entered value.

5.3 Vented-box simulation

When vented-box parameters have been identified, the script <Simulate Vented Box.sce> can be use to simulate the system frequency response and check this response with the free-field response computed from pressure measurements.

Input parameters of this script are shown in figure 6.

The user have to enter the following parameters:
- the directory of the pressure measurement file,
- the name of the pressure measurement file,
- the loudspeaker parameters when mounted in enclosure $R_e, f_{\text{so}}$, $Q_{\text{eo}}, Q_{\text{mo}}, V_{\text{as}}$,
- the Q enclosure losses $Q_{l}, Q_{a}, Q_{p}$,
- the port characteristics $f_p$, $h$,
- the parameters related to enclosure volume $V_e$, $\alpha$,
- the parameters related to cone excursion computation $P_{\text{as}}, S_d$,
- the plots parameters $F_{\text{min}}, F_{\text{max}}, N_{\text{bp}}$,
- the frequency range for the scaling of the free-field measurements $db_m$ to frequency $F_M$.

The free-field amplitude response must be scaled to be superimposed on the simulated response. For this, the script minimizes the mean differences of the two responses from frequency where amplitude is $db_m$ to frequency $F_M$.

6 A typical application: the Onken enclosure

This section shows an example of vented-box parameters identification: the Onken enclosure with an Altec 416-8A loudspeaker.

Old Altec 416-8A speakers need to have the surround cleaned before they can be used. It consists to remove dirt and surplus impregnation. For that, one can use a small brush and acetone cleaning solvent. Figure 7 shows the process of cleaning and the result on a small sector of the surround.
The effect of cleaning the surround is to decrease the resonant frequency of the driver as well as the mechanical losses. Figure 8 shows the impedance curve before and after cleaning.

Once the speaker has been cleaned, it can be measured. We get the following parameters for speaker number 24851:

<table>
<thead>
<tr>
<th>$R_e$ (Ω)</th>
<th>$f_s$ (Hz)</th>
<th>$Q_{es}$</th>
<th>$Q_{ms}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.674</td>
<td>22.66</td>
<td>0.258</td>
<td>7.68</td>
</tr>
</tbody>
</table>

The Onken enclosure was built following the recommendations by Jean Hiraga described in [8]. The net internal estimated volume is 273.5 liters.

Five configurations have been measured:

- configuration A : empty enclosure (see figure 9);
- configuration B : lining material placed only on lateral internal walls;
- configuration C : lining material on all walls;
- configuration D : enclosure with one of the six vents blocked;
- configuration E : enclosure with two of the six vents blocked.

Residuals ajustement of configuration A are depicted figure 10. Both amplitude and phase measurements have been used to identify the vented-box parameters, from 5 Hz to 100 Hz.

Residuals are in Ohm for magnitude and degrees for phase. Identified parameters are:

<table>
<thead>
<tr>
<th>$q$</th>
<th>$f_{sv}$ (Hz)</th>
<th>$Q_{es}$</th>
<th>$Q_{ms}$</th>
<th>$Q_l$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.900</td>
<td>21.50</td>
<td>0.272</td>
<td>8.093</td>
<td>$+\infty$</td>
</tr>
<tr>
<td>$Q_\alpha$</td>
<td>$Q_p$</td>
<td>$h$</td>
<td>$\alpha$</td>
<td>$f_p$ (Hz)</td>
</tr>
<tr>
<td>63</td>
<td>45</td>
<td>1.862</td>
<td>2.517</td>
<td>40.03</td>
</tr>
</tbody>
</table>

These values lead to some remarks:

- The acoustic mass ratio $q$ is lower than 1. This is not surprising because the loudspeaker was measured in free-air where the total air load mass is that of a single face.
- More surprising is the value $Q_l$ of leakage losses who is very high and can be considered as infinite.
The identified value of the system compliance ratio $\alpha$ leads to compute the speaker $V_a$ as
$\quad V_a = \alpha \beta V_b(l)$

Taking $\beta = 1$ because case A is for unlined enclosure, we get $V_a = 688 \text{ l}$.

Identified values for case B to case E are:

<table>
<thead>
<tr>
<th>Case</th>
<th>$q$</th>
<th>$f_{so}$</th>
<th>$Q_l$</th>
<th>$Q_a$</th>
<th>$Q_p$</th>
</tr>
</thead>
<tbody>
<tr>
<td>B</td>
<td>0.884</td>
<td>21.30</td>
<td>+$\infty$</td>
<td>29</td>
<td>38</td>
</tr>
<tr>
<td></td>
<td>$h$</td>
<td>$\alpha$</td>
<td>$f_p(hz)$</td>
<td>$\beta$</td>
<td>$\beta V_b(l)$</td>
</tr>
<tr>
<td></td>
<td>1.787</td>
<td>2.488</td>
<td>38.06</td>
<td>1.012</td>
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Results are consistent with those expected:

- The absorption losses $Q_a$ decrease when lining material is placed inside the enclosure, going from 63 to 29.
- $\beta$ increases the more the enclosure is filled (the apparent volume goes from 273.5 liters up to 287 liters).
- The process of blocking one or two ports reduces significantly the resonant frequency $f_p$.

We can also remark that:

- The acoustic mass ratio $q$ decreases as the enclosure is filled.
- Blocking the vents increase the $\beta$ coefficient.

The last point is maybe due to the fact that the vents were blocked with acoustic foam.

The next step was to compute the free-field frequency response. The pressure inside the enclosure was measured with REW and a calibrated Dayton EMM6 microphone.

The figure 11 shows the measurement with no smoothing for case A.

It can be seen that the measurement is smooth up to 150 hz. From that point, stationary waves inside the enclosure distort significantly the response.

The simulation of the response in case A is depicted figure 12. This figure superimposes the theoretical response calculated with the identified parameters and the measured response in 1/12 octaves transformed according to the equations of section 4.

One can note the very good prediction of the theoretical model. The highest deviation between the theoretical and measured responses is less than 0.6 dB.

The measured group delay broadly follows the theoretical one. The deviation below 20 Hz is probably due to the cutoff frequency of the microphone which is 18 hz.
The response of the speaker in case A leads to a 35.2 Hz cutoff frequency and a peak of 3.9 dB at 46.2 Hz.

Curves in figure [13] shows the case E. Blocking 2 vents on 6 has a very beneficial effect on the frequency response. The peak amplitude is no longer than 0.28 dB and the cutoff frequency goes down to 29.8 Hz.

Once the parameters have been estimated, one can identify a new set of parameters using the script [MeaVented-Box 2.sce] with the identified value of $Q_{mo}$. This leads to compute two new values of $q$ with:

$$q_1 = \left(\frac{f_{so}}{f_s}\right)^2$$

(26)

$$q_2 = \left(\frac{Q_{es}}{Q_{so}}\right)^2$$

(27)

Case E leads to $q_1 = 0.86$ and $q_2 = 0.82$. These values are close to the initial value of $q$ which is 0.86. This shows the consistency of the estimation.

The configuration E was retained to listen to these speakers.

**Conclusion**

This article presents Scilab scripts to identify the vented-box parameters and compute the free-field system frequency response from measurement of the pressure within the enclosure.

The following conclusions can be drawn from this study:

- It is not possible to simultaneously identify $Q_{mo}, Q_i, Q_a, Q_p$ from impedance measurements. However when the Thiele and Small parameters of the loudspeaker are known, by introducing the mass ratio parameter $q$, it is possible to identify all Q losses factors of the enclosure as well as the system tuning ratio and the system compliance ratio.

- To increase accuracy of the identified parameters, one has to use both magnitude and phase of the impedance measurements.

- Enclosure leakage losses are accepted to be dominant in vented-box enclosures. This was not the case in this study where leakage losses were found to be neglectable.

- The simulation of the system frequency response with the identified parameters proved to be very close to the the free-field system response. Variance analysis shows that the accuracy of the simulated response is better than 0.6 dB.

- The estimation of the vented-box parameters from the impedance measurements allows effective fine-tuning of a such loudspeaker system.

**A The least square estimation method**

This section summarizes some properties of the least square method. More details can be found in reference [9].

Let $z$ a vector of $m$ observations and $x$ a vector of $p$ variables. Let suppose that the measurement equation is linear, so that we get:

$$Jx = z + \epsilon$$

(28)

where $J$ is a matrix of $m$ rows and $p$ columns. This model includes the hypothesis:

- The matrix $J$ is of rank $p$

- The measurement noise has zero mean : $E(\epsilon) = 0$

- The measurement noise covariance matrix is : $V(\epsilon) = E(\epsilon\epsilon^T) = \sigma^2 \Gamma$

where $\Gamma$ is a known positive definite square matrix of order $m$.

The least square estimation of $x$, noted $\hat{x}$ is given by:

$$\hat{x} = (J^T\Gamma^{-1}J)^{-1}J^T\Gamma^{-1}z$$

(29)

Let us note $A = J^T\Gamma^{-1}J$. The variance of the estimator $\hat{x}$ is:

$$V(\hat{x}) = \sigma^2 A^{-1}$$

(30)

It can be demonstrate that the least square estimator is the minimum variance estimator within the class of unbiased linear estimator.

Let us suppose now that the exact values of $p-1$ variables of $x$ are known. The estimation of the unknown component $x_i$ is obtain by regression through the origin. We can write:

$$z' = J_i x_i + \epsilon$$

(31)

with:

$$z' = z - \sum_{j \neq i} J_j x_j$$

(32)

and where vector $J_i$ is the $i$th column of matrix $J$. The variance of $x_i$ is:

$$V_i = \frac{\sigma^2}{J_i^T\Gamma^{-1}J_i}$$

(33)

Let us return to the normal situation where all variables of $x$ is to be estimated. It can be demonstrate that the least square estimator satisfies:

$$\forall i \ V(\hat{x}_i) > \frac{\sigma^2}{J_i^T\Gamma^{-1}J_i}$$

(34)

The ratio $V(\hat{x}_i)/V_i$ is called the variance inflation factor of the parameter $i$ and is written $VIF_i$. This
number is always greater than or equal to 1. The variance inflation factor of a parameter can be very large. It is related to the lack of observability of the parameter in question.

Let us perform a non-singular linear transformation in parameter space $x$ so that:

$$\mathbf{x} = K \mathbf{x}$$  \hspace{1cm} (35)

where $K$ is a diagonal matrix with:

$$k_i = \sqrt{J_i^T \Gamma^{-1} J_i} \hspace{1cm} (36)$$

Equation (28) leads to:

$$JK^{-1} \mathbf{X} = \mathbf{z} + \mathbf{\epsilon} \hspace{1cm} (37)$$

Thus:

$$\mathbf{J} \mathbf{X} = \mathbf{z} + \mathbf{\epsilon} \hspace{1cm} (38)$$

with $\mathbf{J} = JK^{-1}$. The matrix $\mathbf{A}$ related to the least square estimation of $\mathbf{x}$ is written:

$$\mathbf{A} = \mathbf{J}^T \Gamma^{-1} \mathbf{J} = K^{-1} A K^{-1} \hspace{1cm} (39)$$

All of the diagonal elements of this matrix are equal to 1. It is said that this matrix has the form of a correlation matrix.

Diagnosing approximative collinearity is based on the spectral analysis of $\mathbf{A}$. Indeed, if $s'$ (for $i$ between 1 and $p$) is an orthonormal basis composed of eigenvectors of $\mathbf{A}$, and the vector $s_j$ is associated with the eigenvalue $\mu_j$, then:

$$\text{FIV}_i = \sum_j s_{ij}^2 \mu_j \hspace{1cm} (40)$$

where $s_{ij}$ is the $i$th component of the vector $s_j$. This relation shows that small eigenvalues $\mu_j$ leads to large variance inflation factors.

Let us note $\mu_{\text{min}}$ the smallest eigenvalue of $\mathbf{A}$. If $p\mu_{\text{min}}$ is less than $10^{-3}$, the lack of observability is said to be severe. If its value is between $10^{-3}$ and $10^{-2}$, the lack of observability is said to be strong.

The choice of the parameter $p\mu_{\text{min}}$ comes from the relation:

$$\max_i \text{FIV}_i \geq \frac{1}{p\mu_{\text{min}}} \hspace{1cm} (41)$$

Thus for severe lack of observability, there are variance inflation factors greater than 1000.

The eigenvectors corresponding to the small eigenvalues give the coefficients of the linear combination of normalized parameters which are most likely to be poorly observed. On the other hand, the parameters that do not appear in these combinations are well observed.

### B Links to the Scilab scripts

- [http://www.volucres.fr/AudioHighEnd/resources/SciAudioBoxEn/SciAudioBox.sci](http://www.volucres.fr/AudioHighEnd/resources/SciAudioBoxEn/SciAudioBox.sci)
- [http://www.volucres.fr/AudioHighEnd/resources/SciAudioBoxEn/Measure-Vented-Box-1.sce](http://www.volucres.fr/AudioHighEnd/resources/SciAudioBoxEn/Measure-Vented-Box-1.sce)
- [http://www.volucres.fr/AudioHighEnd/resources/SciAudioBoxEn/Measure-Vented-Box-2.sce](http://www.volucres.fr/AudioHighEnd/resources/SciAudioBoxEn/Measure-Vented-Box-2.sce)
- [http://www.volucres.fr/AudioHighEnd/resources/SciAudioBoxEn/Simulation-Vented-Box.sce](http://www.volucres.fr/AudioHighEnd/resources/SciAudioBoxEn/Simulation-Vented-Box.sce)

### C Links to impedance and pressure measurements of case E

- [http://www.volucres.fr/AudioHighEnd/resources/SciAudioBoxEn/Vented-box-Impedance.txt](http://www.volucres.fr/AudioHighEnd/resources/SciAudioBoxEn/Vented-box-Impedance.txt)
- [http://www.volucres.fr/AudioHighEnd/resources/SciAudioBoxEn/Vented-box-Impedance.lim](http://www.volucres.fr/AudioHighEnd/resources/SciAudioBoxEn/Vented-box-Impedance.lim)
- [http://www.volucres.fr/AudioHighEnd/resources/SciAudioBoxEn/Internal-Pressure.txt](http://www.volucres.fr/AudioHighEnd/resources/SciAudioBoxEn/Internal-Pressure.txt)
- [http://www.volucres.fr/AudioHighEnd/resources/SciAudioBoxEn/Internal-Pressure.mdat](http://www.volucres.fr/AudioHighEnd/resources/SciAudioBoxEn/Internal-Pressure.mdat)

### References


[6] LIMP program for the loudspeaker impedance measurement and loudspeaker parameters estimation : [http://www.artalabs.hr](http://www.artalabs.hr)


