Least Squares Estimation of Vented-Box Parameters from Impedance Measurements

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Abstract

This paper describes Scilab scripts that are used to identify vented-box parameters from impedance measurements. These scripts use non-linear least squares optimization to estimate the enclosure losses (port, leakage and absorption losses), the system tuning ratio and the system compliance ratio. Additionally, these scripts calculate the free-field frequency response based on measurement of the acoustical pressure within the enclosure.

Nomenclature

α System compliance ratio $= V_{as}/V_{ab}$

β Apparent volume to net enclosure volume ratio $= V_{ab}/V_b$

$\overline{G}(S)$ System response function

$\overline{P}_b$ Pressure function inside the enclosure

$\overline{P}_e$ Free-field pressure function

$\overline{p}_g$ Acoustic pressure generator $= Bl\overline{U}_g/S_dR_e$

$\overline{q}_b$ Enclosure volume velocity

$\overline{q}_d$ Loudspeaker volume velocity

$\overline{q}_l$ Leakage volume velocity

$\overline{q}_p$ Vent volume velocity

$\overline{U}_g$ Electric source output voltage

$\overline{Z}(S)$ Impedance function of the loudspeaker mounted in enclosure

ρ Air density

B Magnetic flux density in driver air gap

c Speed of sound

C_{ab} Acoustic compliance of air in enclosure

C_{as} Acoustic compliance of driver suspension

C^*_{eo} Electrical capacitance due to driver mass $= M^*_{ao}S_d^2/(Bl)^2$

C^*_{ep} Electrical capacitance due to vent mass $= M^*_{ap}S_d^2/(Bl)^2$

f Current frequency

f_p Resonance frequency of vented enclosure $= 1/2\pi\sqrt{M^*_{ap}C_{ab}}$

f_s Resonance frequency of driver $= 1/2\pi\sqrt{M^*_{as}C_{as}}$

f_{so} Resonance frequency of driver mounted in enclosure $= 1/2\pi\sqrt{M^*_{ao}C_{as}}$

h System tuning ratio $= f_p/f_{so}$

l Effective length of voice coil conductor

L_{eb} Electrical inductance due to enclosure compliance $= C^*_{ab}S_d^2/(Bl)^2$

L_{es} Electrical inductance due to driver suspension compliance $= C^*_{as}S_d^2/(Bl)^2$

M^*_{ao} Acoustic mass of driver diaphragm including air load when mounted in enclosure

M^*_{ap} Acoustic mass of port including air load

M^*_{as} Acoustic mass of driver diaphragm including air load

q Driver acoustic mass ratio $= M^*_{as}/M^*_{ao}$

Q_a Enclosure Q at f_p resulting from absorption losses $= 1/2\pi f_p C_{ab}R_{ab}$

Q_l Enclosure Q at f_p resulting from leakage losses $= 2\pi f_p C_{ab}R_{al}$

Q_p Enclosure Q at f_p resulting from vent frictional losses $= 1/2\pi f_p C_{ab}R_{ap}$

Q_{eo} Electric driver losses at f_{so} when mounted in enclosure $= 1/2\pi f_{so}C_{as}R_{as}$

Q_{es} Electric driver losses at f_s

Q_{mo} Mechanical driver losses at f_{so} when mounted in enclosure $= 1/2\pi f_{so}C_{as}R_{ae}$

Q_{ms} Mechanical driver losses at f_s

Q_{to} Total driver Q at f_{so} resulting from electric and mechanical driver losses when mounted in enclosure

r Distance from the loudspeaker

R_{ab} Acoustic resistance of enclosure losses caused by internal energy absorption

R_{ae} Acoustic resistance of driver electric losses

R_{al} Acoustic resistance of enclosure losses caused by leakage

R_{ap} Acoustic resistance of vent losses

R_{as} Acoustic resistance of driver suspension losses

R_{eb} Electrical resistance due to enclosure absorption resistance $= (Bl)^2/S_d^2 R_{ab}$

R_{el} Electrical resistance due to enclosure leakage resistance $= (Bl)^2/S_d^2 R_{al}$

R_{ep} Electrical resistance due to resistance of vent losses $= (Bl)^2/S_d^2 R_{ap}$

R_{es} Electrical resistance due to driver suspension resistance $= (Bl)^2/S_d^2 R_{as}$

R_e DC resistance of driver voice coil

S Normalized laplace variable $= s/2\pi f_{so}$

s Laplace variable

S_d Effective area of driver diaphragm

V_b Net internal volume of enclosure

V_{ab} Volume which represents the acoustic compliance of the enclosure $= \rho c^2 C_{ab}$

V_{as} Volume of air having same acoustic compliance as driver suspension $= \rho c^2 C_{as}$

Introduction

In order to be able to accurately simulate the low-frequency response of a vented-box loudspeaker system, it is necessary to know the enclosure losses, the Thiele and Small parameters of the driver, the resonant frequency of the vented enclosure and the volume which represents the acoustic compliance of the enclosure. Determination of these parameters is not without problems.

- Three kinds of enclosure losses have to be taken into account : the leakage losses Q_l, the absorption losses Q_a and the vent losses Q_p. Magnitudes of these losses have been determined by Small in [2]. Typical values for Q_p are in range 50-100. Typical value for Q_a is 100 for unlined enclosures and between 30-80 when lining material is placed on the enclosure. Small consider that leakage losses are the most significant giving Q_l values between 5 and 20. These recommendations have been taken by WinISD pro [5] which use Q_l of 10 by default.

- The Thiele and Small parameters of the loudspeaker can be taken from the manufacturer. These parameters are $R_e, f_s, Q_{es}, Q_{ms}, V_{as}$. However, production tolerance are such that it is preferable to measure them, and it is imperative to do it in case of old loudspeakers. The measurement of the loudspeaker should be such that it is loaded by the same air load mass as when it is mounted in the vented enclosure. The best way to achieve that is to mount the loudspeaker in a baffle such as the one recommended in the IEC 268 standard. However the size of this baffle for a 15 inch loudspeaker is very huge and it is more convenient to measure the driver free-air. This leads to different air load mass and the resonance frequency of driver mounted in the enclosure f_{so} will differ from the free-air resonance frequency f_s. As a consequence the values Q_{eo}, Q_{mo} will also differ from Q_{es}, Q_{ms}.

  Another problem is related to the measure of the V_{as} parameter. The easiest and quicker method is the Delta mass method. However a single measurement with such a technique cannot provide better accuracy than 5 %.

- The resonant frequency of the vented enclosure f_p is computed from port characteristics. Port end-correction depends on the number of vents, their shapes and their mounting (flush-mounted or not). Different empirical formulas are used for calculating this correction. None of these formulas is very accurate.

- The volume which represents the acoustic compliance of the enclosure will differ from the net internal volume enclosure because of the lining material place on it. The value of the β coefficient can vary between 1, when unfilled, to 1.4, when totally filled. It is not possible to give accurate value of β as it depends on the actual material used and where it is placed.

As we can see, parameters needed to simulate the low frequency response of a vented-box, which are $Q_l, Q_a, Q_p, f_{so}, Q_{eo}, Q_{mo}, f_p, \beta$, are not easily determined.

The purpose of this paper is to study how these parameters can be identified and provide Scilab scripts to do it.

1 Acoustical equivalent circuit of a vented-box loudspeaker system

Referring to small paper [2] the acoustical analogous circuit of a vented-box is presented figure 1.

Using overline notation for Laplace function, the system response fonction is given by :

$$\overline{G}(S) = \frac{a_4 S^4 + b_3 S^3}{a_4 S^4 + a_3 S^3 + a_2 S^2 + a_1 S + a_0} \qquad (1)$$

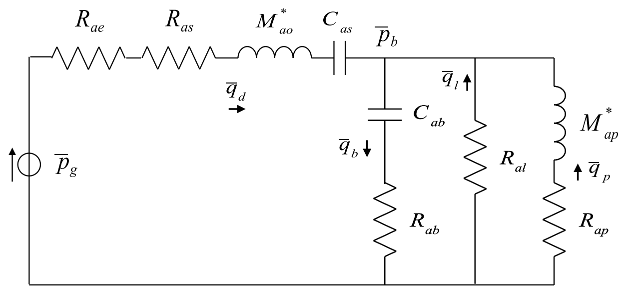

Figure 1: Acoustical analogous circuit of vented-box

The coefficients are :

$$a_0 = h^3(1 + Q_p^{-1}Q_l^{-1}) \tag{2}$$
$$a_1 = h^3Q_{to}^{-1}(1 + Q_p^{-1}Q_l^{-1}) + h^2\alpha Q_p^{-1} + h^2(Q_p^{-1} + Q_a^{-1} + Q_l^{-1} + Q_p^{-1}Q_a^{-1}Q_l^{-1}) \tag{3}$$
$$a_2 = h^3(1 + Q_p^{-1}Q_l^{-1}) + h^2Q_{to}^{-1}(Q_p^{-1} + Q_a^{-1} + Q_l^{-1} + Q_p^{-1}Q_a^{-1}Q_l^{-1}) + h(\alpha(1 + Q_p^{-1}Q_a^{-1}) + 1 + Q_a^{-1}Q_l^{-1}) \tag{4}$$
$$a_3 = h^2(Q_p^{-1} + Q_a^{-1} + Q_l^{-1} + Q_p^{-1}Q_a^{-1}Q_l^{-1}) + hQ_{to}^{-1}(1 + Q_a^{-1}Q_l^{-1}) + \alpha Q_a^{-1} \tag{5}$$
$$a_4 = h(1 + Q_a^{-1}Q_l^{-1}) \tag{6}$$
$$b_3 = h^2Q_p^{-1}(1 + Q_a^{-1}Q_l^{-1}) \tag{7}$$

The system response is a fourth-order high-pass filter function depending only on $f_{so}, Q_{to}, Q_l, Q_a, Q_p, h, \alpha$.

The effects of enclosure losses on response is shown on figure 2 where the lossless curve is a B-4 aligned vented-box loudspeaker system.

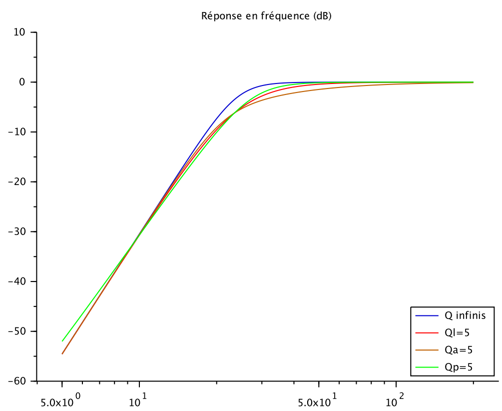

Figure 2: Effects of Q losses on response of a vented-box loudspeaker system

It can be seen that enclosure losses have a significant effect whose main drawback is to decrease the cutoff frequency. This leads to kept these losses to minium. As the absorption losses Q_a increases when lining material is placed in the enclosure, a minimal amount of acoustic foam must be used. As a consequence β will not generally be higher than 1.1.

The review of the coefficients of the transfert function (1) shows that the system response does not depend directly on V_{as} but only on α, the system compliance ratio :

$$\alpha = \frac{V_{as}}{\beta V_b} \tag{8}$$

From this equation we can conclude that :

- An accurate measurement of V_{as} is not needed because the true value of β is unknown. In most case, the V_{as} value can be taken directly from the manufacturer of the loudspeaker.
- The internal volume V_b can be adjusted to reach the target value of α for the true values of V_{as} and β. It is therefore possible to get the exact desired system response even if true values of V_{as} and β are initially unknown. This can be done by building the enclosure box with an initial higher volume which will be decreased during the optimization process.

2 Electrical equivalent circuit of a vented-box loudspeaker system

The electrical equivalent circuit of the vented-box loudspeaker system is formed by taking the dual of the acoustic circuit of figure 1 and converting each element to its electrical equivalent. We get the circuit of figure 3.

From this figure we can calculate the electrical impedance function :

$$\overline{Z}(S) = R_e\frac{b_4S^4 + b_3S^3 + b_2S^2 + b_1S + b_0}{a_4S^4 + a_3S^3 + a_2S^2 + a_1S + a_0} \tag{9}$$

The coefficients are :

$$a_0 = h^3(1 + Q_p^{-1}Q_l^{-1}) \tag{10}$$
$$a_1 = h^3Q_{mo}^{-1}(1 + Q_p^{-1}Q_l^{-1}) + h^2\alpha Q_p^{-1} + h^2(Q_p^{-1} + Q_a^{-1} + Q_l^{-1} + Q_p^{-1}Q_a^{-1}Q_l^{-1}) \tag{11}$$
$$a_2 = h^3(1 + Q_p^{-1}Q_l^{-1}) + h^2Q_{mo}^{-1}(Q_p^{-1} + Q_a^{-1} + Q_l^{-1} + Q_p^{-1}Q_a^{-1}Q_l^{-1}) + h(\alpha(1 + Q_p^{-1}Q_a^{-1}) + 1 + Q_a^{-1}Q_l^{-1}) \tag{12}$$
$$a_3 = h^2(Q_p^{-1} + Q_a^{-1} + Q_l^{-1} + Q_p^{-1}Q_a^{-1}Q_l^{-1})$$

$$+ hQ_{mo}^{-1}(1 + Q_a^{-1}Q_l^{-1}) + \alpha Q_a^{-1} \quad (13)$$

$$a_4 = h(1 + Q_a^{-1}Q_l^{-1}) \quad (14)$$

$$b_0 = h^3(1 + Q_p^{-1}Q_l^{-1}) \quad (15)$$

$$b_1 = h^3 Q_{to}^{-1}(1 + Q_p^{-1}Q_l^{-1}) + h^2\alpha Q_p^{-1} + h^2(Q_p^{-1} + Q_a^{-1} + Q_l^{-1} + Q_p^{-1}Q_a^{-1}Q_l^{-1}) \quad (16)$$

$$b_2 = h^3(1 + Q_p^{-1}Q_l^{-1}) + h^2 Q_{to}^{-1}(Q_p^{-1} + Q_a^{-1} + Q_l^{-1} + Q_p^{-1}Q_a^{-1}Q_l^{-1}) + h(\alpha(1 + Q_p^{-1}Q_a^{-1}) + 1 + Q_a^{-1}Q_l^{-1}) \quad (17)$$

$$b_3 = h^2(Q_p^{-1} + Q_a^{-1} + Q_l^{-1} + Q_p^{-1}Q_a^{-1}Q_l^{-1}) + hQ_{to}^{-1}(1 + Q_a^{-1}Q_l^{-1}) + \alpha Q_a^{-1} \quad (18)$$

$$b_4 = h(1 + Q_a^{-1}Q_l^{-1}) \quad (19)$$

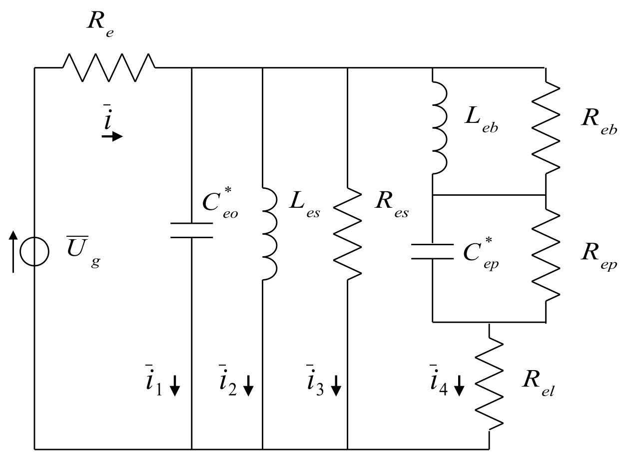

Figure 3: Electrical circuit of vented-box

Equation (9) of the impedance function depends on $R_e, f_{so}, Q_{eo}, Q_{mo}, Q_l, Q_a, Q_p, h, \alpha$.

3 Parameters identification from impedance measurement

Apart from R_e which can easily be measure with an ohmmeter, one might be tempted to try to identify all parameters from which the impedance function depends.

To do so, as a first step, we have computed simulated impedance values of a vented-box loudspeaker system with the following parameters :

f_{so}(hz)	Q_{eo}	Q_{mo}	Q_l	Q_a	Q_p	h	α
23	0.3	8	20	15	10	1.5	2

Then, using least squares estimation technic, we have identify all of these parameters from the simulated measurements. The identification process gives the following values :

f_{so}(hz)	Q_{eo}	Q_{mo}	Q_l
23	0.3	7.78	18.10
Q_a	Q_p	h	α
15.60	10.26	1.4998	2.0001

The figure 4 depicts residuals, i.e. differences between the measurements values and the fitted values provided by the model.

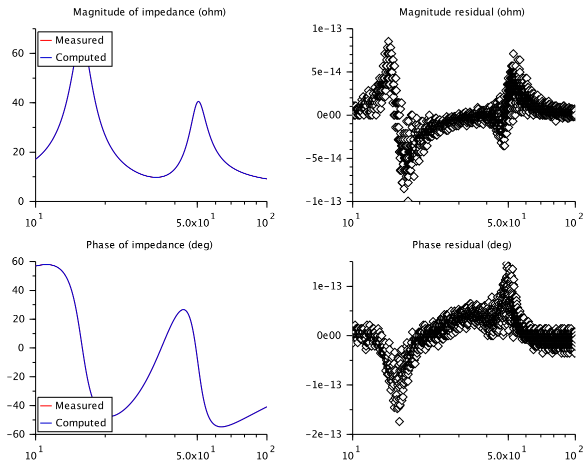

Figure 4: Impedance curves and residual adjustements

As expected, residuals are zero because we used the same model to simulate the measurements and identify the parameters. However, the solution does not converge to the initial settings. Apart from f_{so} and Q_{eo}, all others parameters differ.

An observability check can be carried out with the eigenvalues of the normal equation matrix (see annexe A).

The smallest eigenvalue is 2.2 10^{-12} which indicates that this matrix is singular and that all parameters cannot be identified. The eigenvector coordinates associated with this eigenvalue are :

f_{so}	Q_{eo}	Q_{mo}	Q_l
2e-7	1e-6	-0.30	-0.63
Q_a	Q_p	h	α
0.57	0.43	-1.9e-2	4.2e-3

The significant coordinates relate on the parameters Q_{mo}, Q_l, Q_a, Q_p, and to a lesser extent on h, α. The coordinates associated with f_{so} and Q_{eo} are almost zero. We can conclude that the lack of observability does not concern f_{so} and Q_{eo} but Q_{mo}, Q_l, Q_a, Q_p. This is consistent with what we get from estimated values. Estimations give good values of f_{so} and Q_{eo}, close values of h, α and bad values of Q_{mo}, Q_l, Q_a, Q_p.

Signs of the eigenvector coordinates provide the way the estimated parameters vary. It can be seen that a decrease of Q_{mo} will result in a decrease of Q_l and an increase of Q_a and Q_p.

The reason for this can be clearly understood from the electric circuit of figure 3. The resistance R_{es} which define Q_{mo} is in parallel with the resistances R_{el}, R_{eb}, R_{ep} which respectively define Q_l, Q_a, Q_p. One understand that an increase of R_{es} can be compensated by a decrease of R_{el}, R_{eb}, R_{ep} to finally get the same value of the impedance. The definitions of Q losses factors are consistent with the variations of Q_{mo}, Q_l, Q_a, Q_p previously observed.

Closer examination of the coefficients of the impedance transfert function shows that there is actually one degree of freedom within $Q_{mo}, Q_l, Q_a, Q_p, h, \alpha$. More details can be found in annexe B of reference [10].

We can conclude therefore that it is not possible to identify simultaneously the four parameters Q_{mo}, Q_l, Q_a, Q_p from impedance measurements.

4 Free-field frequency response measurement

The free-field low frequency response measurement of a loudspeaker system is a difficult task. It cannot be performed in a semi-reverberant room because of the modal response of this room. It cannot also be performed in a standard anechoic chamber because most of them have a frequency cutoff greater than 60 hz. There are mainly two technics to measure the free-field low frequency response. The first one, proposed by D.B. Keele in [3], is based on the measurement taken in the near-field outside the enclosure. The second one, proposed by Small in [4], consists to calculate the free-field frequency response from the measurement of the acoustic pressure within the system enclosure.

The first method is more complicated for a vented-box than for a closed box because it needs to sum the near-field measurements of the loudspeaker diaphragm and vent. The summation implies to be vectorial which means that both magnitude and phase have to be considered. The level of the diaphragm and the vent must be adjusted before the response is summed. This adjustment is made according to their respective equivalent diameter. However, this method does not require to know any of the vented-box parameters.

The second method requires only a measurement of the pressure within the enclosure regardless of the number of radiating surfaces. However the vent resonant frequency and the absorption losses need to be known to accurately compute the free-field frequency response. This is the method we use in this paper.

Let us denote $\overline{P}_b$ the measurement of the pressure inside the enclosure. From analysis of the acoustical equivalent circuit of figure 1, the relation between internal pressure and internal volume velocity is :

$$\overline{P}_b = (R_{ab} + \frac{1}{j2\pi f C_{ab}})\overline{q}_b \tag{20}$$

The external pressure outside the enclosure must be computed with the sum of all radiating surfaces (diaphragm, vent and leakage). As we have :

$$\overline{q}_b = \overline{q}_d + \overline{q}_l + \overline{q}_p \tag{21}$$

the external pressure can be simply computed with the internal volume velocity, which makes this method very simple.The external pressure, at a distance r from the enclosure, is therefore given by :

$$\overline{P}_e = \frac{\rho f}{r}\overline{q}_b \tag{22}$$

From definitions of C_{ab} and R_{ab} we get :

$$C_{ab} = \frac{\beta V_b}{\rho c^2} \tag{23}$$

$$\omega R_{ab} C_{ab} = \frac{f}{f_{so}}(hQ_a)^{-1} \tag{24}$$

Combining these equations, we obtain :

$$\overline{P}_e = \frac{\beta V_b 2\pi f_{so}^2}{r}\frac{S^2}{1+S(hQ_a)^{-1}}\overline{P}_b \tag{25}$$

This method is valid as long as the pressure inside the enclosure is uniform and the development of standing wave within the enclosure make it useless. Measurements show that the pressure inside the enclosure becomes noticeably non uniform even below the first standing wave. To improve the validity range of this method, the microphone should be placed near the geometrical center of the enclosure.

5 Description of Scilab scripts

Scilab is an open source software for numerical computation. Scilab can be downloaded from [1] and is available for Window, Linux or Mac. Scilab can be used interactively, by typing commands in the console window. Scilab provides also a powerful editor, Scinotes, to edit scripts. Names of script have extension **.sce** or **.sci**. The files having extension **.sci** contains Scilab functions. Executing them loads the functions into the Scilab environnement. The files having extension **.sce** contains executables.

Scilab scripts describe in this paper are <*SciAudioBox.sci*>, <*Measure Vented-Box 1.sce*>, <*Measure Vented-Box 2.sce*> and <*Simulate Vented Box.sce*>.

The <*SciAudioBox.sci*> is the library which contains the main functions used by others scripts. It must be run once before execution of one of the other scripts.

<*Measure Vented-Box 1.sce*> and <*Measure Vented-Box 2.sce*> scripts read impedance measurement from a file to identify vented-box parameters. Impedance measurements must be saved in an ASCII text file. Non-numeric lines are ignored. Data lines must begin with the frequency in hz, then the impedance magnitude in Ohm and finally the phase in degrees.

<*Simulate Vented Box.sce*> script read pressure measurement from a file to compute the free-field frequency response. Format of this file is identical to that of the impedance, expect that magnitude is in dB.

Softwares like ARTA [6] of REW [7] can be used to measure impedance and acoustic pressure. The exported file formats by these softwares are compatible with the one used by the Scilab scripts.

5.1 Identification of vented-box parameters with a known loudspeaker

In section 3 we have seen that it is not possible to identify simultaneously Q_{mo}, Q_l, Q_a, Q_p from impedance measurements. As a consequence the value of Q_{mo} must be obtained through other means. Unfortunately, the measurement of the loudspeaker gives Q_{ms} but not Q_{mo}. Let us denote q the acoustic mass ratio. From definitions of f_{so}, Q_{eo}, Q_{mo} and f_s, Q_{es}, Q_{ms} we get :

$$f_{so} = f_s\sqrt{q} \ , \ Q_{eo} = \frac{Q_{es}}{\sqrt{q}} \ , \ Q_{mo} = \frac{Q_{ms}}{\sqrt{q}}$$

As f_{so} and Q_{eo} are well observed from impedance measurements, it is obvious that knowing f_s and Q_{es} leads to well estimate q. It is therefore clear that identifying q instead of f_{so}, Q_{eo}, Q_{mo} will get rid of the lack of observability and make all parameters fully estimated.

This is the purpose of the script <*Measure Vented-Box 1.sce*>.

The figure 5 shows the input parameters of this script.

The user have to enter the following parameters :

- the directory of the impedance measurement file,
- the name of the impedance measurement file,
- the loudspeaker parameters R_e, f_s, Q_{es}, Q_{ms},
- the net volume of enclosure V_b and the volume of air having same compliance as loudspeaker suspension V_{as},
- the initial guess of $q, Q_l, Q_a\ Q_p, h, \alpha$.

```
// Identification of vented-box parameters with a known loudspeaker.
// Know parameters are : Fs, Qes, Qms of the loudspeaker.
// Unknown parameters to be identified are : q, Ql, Qa, Qp, h, a (alpha) and
// possibly Re.

// Input data -----------------------------------------------------------------

// Directory of impedance measurements file
d = home+"/Sites/Audio High End/SciAudioBox/Mesures/";

// Impedance measurement used for identification
// TypAjust = 1 : Magnitude
// TypAjust = 2 : Phase
// TypAjust = 3 : Magnitude and phase
TypAjust = 3 ;

// Identification of Re, the resistance of the voice coil
// ReAjust = 1 : Re identified
// ReAjust = 0 : Re not identified
ReAjust = 0;

// Name of the impedance measurement file.
// Format of the file : frequency (hz), amplitude (ohm), phase (deg)
fic = "Simu Event.txt";

// Maximum frequency use for identification
Fqmax = 100 ;

// Loudspeaker parameters
Re = 6.5 ;   // Resistance of voice coil (ohm)
Fs = 23 ;    // Resonance frequency (hz)
Qes = 0.3 ;  // Electric losses
Qms = 8 ;    // Mechanical losses

// Other loudspeaker parameters
// These parameters are not needed for the identification process.
// There are used only to compute b (beta) and Vab=b.Vb
Vas = 579.58 ; // Volume of air having same compliance as driver suspension (l)
Vb = 273.5 ;   // Net volume of enclosure (l)

// Initial values of unknown parameters
q = 1;     // Driver acoustic mass ratio
Ql = 5;    // Leakage losses
Qa = 5;    // Absorption losses
Qp = 5;    // Vent losses
h = 1.7;   // System tuning ratio
a = 3;     // System compliance ratio
```

Figure 5: Input data of Measure Vented-Box 1 script

V_{as} and V_b are not needed for the identification process. They are just used to compute the β parameter and the apparent volume V_{ab} of the enclosure.

If the voice coil resistance R_e is not known, it can also be identified from impedance measurements. To do so, the user have to set the variable *ReAjust* to 1.

The measurements taken into account by the least squares algorithm are all measurement from the begin of the file up to the F_M frequency. This frequency should not be set to heigh but kept to minium. It can be set by trial and error. In most cases the value of 100 hz is suitable.

The parameter *TypAjust* defines what kind of measurement are taken into account by the identification process. It can be : the magnitude of the impedance, the phase of the impedance, or both the magnitude and phase of the impedance.

Outputs of the script are :

- the least squares algorithm return code, which is 1 when identification succeeded,
- the standard deviation and the maximum values of magnitude and phase residuals,
- the vented-box parameters $R_e, q, Q_l, Q_a, Q_p, h, \alpha$,
- others parameters $f_{so}, Q_{eo}, Q_{mo}, f_p, \beta, V_{ab}$,
- the plots of impedance magnitude and phase as well as residuals.

Running this script with the simulated test case of section 3 leads to perfectly identify parameters used for the simulation.

To quantify the level of observability of identified parameters one can compute the value $p\mu_{min}$ where p is the number of identified parameters and μ_{min} the smallest eigenvalue of the least squares matrix (see annexe A). The higher the value, the higher the parameters are identified. The use of only the magnitude of the impedance, leads to a value of 0.02 while the use of both magnitude and phase leads to 0.12. It is therefore recommended to use both magnitude and phase.

5.2 Identification of vented-box parameters with an unknown loudspeaker

When Thiele and Small parameters of the loudspeaker are unknown and cannot be measured, it is always possible to identify the vented-box parameters with a given value of Q_{mo}. This is the purpose of the script <*Measure Vented-Box 2.sce*>. Input parameters of this script are not very different from the previous one. Instead entering T/S parameters of the loudspeaker, the user has just to enter the known value of Q_{mo}. The accuracy of the results is of course directly related to the accuracy of the entered value.

5.3 Vented-box simulation

When vented-box parameters have been identified, the script <*Simulate Vented Box.sce*> can be use to simulate the system frequency response and check this response with the free-field response computed from pressure measurements.

Input parameters of this script are shown in figure 6.

The user have to enter the following parameters :

- the directory of the pressure measurement file,
- the name of the pressure measurement file,
- the loudspeaker parameters when mounted in enclosure $R_e, f_{so}, Q_{eo}, Q_{mo}, V_{as}$,
- the Q enclosure losses Q_l, Q_a, Q_p,
- the port characteristics f_p, h,
- the parameters related to enclosure volume V_b, α,
- the parameters related to cone excursion computation P_{as}, S_d,
- the plots parameters F_{min}, F_{max}, Nbp,
- the frequency range for the scaling of the free-field measurements db_m, F_M.

The free-field amplitude response must be scaled to be superimposed on the simulated response. For this, the script minimizes the mean differences of the two responses from frequency where amplitude is db_m to frequency F_M.

```
// Vented-box simulation. Compute system frequency response, impedance, cone
// excurion and group delay.

// Données d'entrées ------------------------------------------------------------

// Directory of pressure measurements file
d = home+"/Sites/Audio High End/SciAudioBox/Mesures/";

// Thiele and Small loudspeaker parameters when mounted in enclosure.
Re = 6.5 ;    // Resistance of voice coil (ohm)
Fso = 23 ;    // Resonance frequency (hz)
Qeo = 0.3 ;   // Electric losses
Qmo = 8 ;     // Mechanical losses
Vas = 600;    // Volume of air having same compliance as driver suspension (l)

// Vented-box Q losses
Ql = 20;    // Leakage losses
Qa = 15;    // Absorption losses
Qp = 10;    // Vent frictional losses

// Port characteristics
Fb = 40 ; // Resonance frequency of vented enclosure (hz)
// System tuning ratio
h = Fb/Fso;
h = 1.5;

// Parameters related to enclosure volume
b = 1.;      // Apparent volume to net enclosure volume ratio
Vb = 300;    // Net internal volume of enclosure (l)
// System compliance ratio
a = Vas/(b*Vb);
a = 2.;

// Paramètres related to cone excursion computation
Pas = 0.062 ;      // Generated acoustic power (0.062 Watts  = 100 dB at 1m)
Sd = 825.75241 ;   // Effective area of driver diaphragm (cm^2)

// Name of the pressure measurement file.
// Format of the file : frequency (hz), amplitude (dB), phase (deg)
// ="" if no measurements were performed
fic = "";

// Plot parameters
Fmin = 10 ;     // Minimum frequency (hz)
Fmax = 150 ;    // Maximum frequency (hz)
Nbp = 5000 ;    // Number of points

// Frequency range for free-field measurement amplitude calibration
dbMin = -5;
FRMax = 100;
```

Figure 6: Input data of Simulate Vented-Box script

Outputs of the script are :

- the cutoff frequency,
- the maximum amplitude value of the system frequency response,
- the group delay in ms at 20, 30, 40 and 50 hz,
- plots of impedance, cone excursion, frequency response and group delay.

6 A typical application : the Onken enclosure

This section shows an example of vented-box parameters identification : the Onken enclosure with an Altec 416-8A loudspeaker.

Old Altec 416-8A speakers need to have the surround cleaned before they can be used. It consists to remove dirt and surplus impregnation. For that, one can use a small brush and acetone cleaning solvent. Figure 7 shows the process of cleaning and the result on a small sector of the surround.

The effect of cleaning the surround is to decrease the resonant frequency of the driver as well as the mechanical losses. Figure 8 shows the impedance curve before and after cleaning.

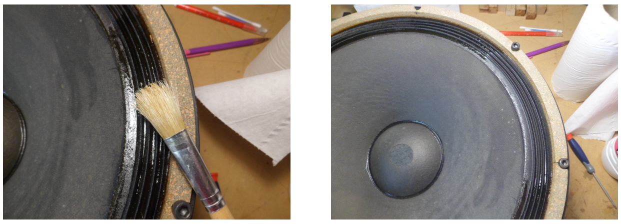

Figure 7: Cleaning surround

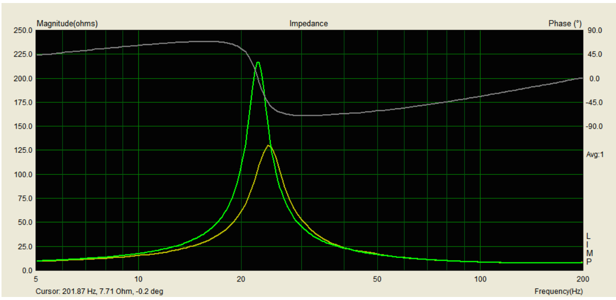

Figure 8: Altec 416 impedance curves

Once the speaker has been cleaned, it can be measured. We get the following parameters for speaker number 24851 :

$R_e(\Omega)$	f_s (hz)	Q_{es}	Q_{ms}
6.674	22.66	0.258	7.68

The Onken enclosure was built following the recommendations by Jean Hiraga described in [8]. The net internal estimated volume is 273.5 liters.

Five configurations have been measured :

- configuration A : empty enclosure (see figure 9);
- configuration B : lining material placed only on lateral internal walls ;
- configuration C : lining material on all walls ;
- configuration D : enclosure with one of the six vents blocked ;
- configuration E : enclosure with two of the six vents blocked.

Residuals ajustement of configuration A are depicted figure 10. Both amplitude and phase measurements have been used to identify the vented-box parameters, from 5 hz to 100 hz.

Figure 9: Empty Onken enclosure

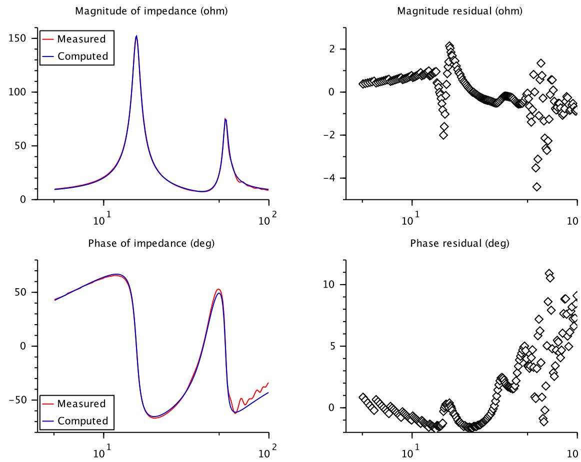

Figure 10: Impedance curves and residual adjustements of case A

Residuals are in Ohm for magnitude and degrees for phase. Identified parameters are :

q	$f_{so}(hz)$	Q_{eo}	Q_{mo}	Q_l
0.900	21.50	0.272	8.093	$+\infty$

Q_a	Q_p	h	α	$f_p(hz)$
63	43	1.862	2.517	40.03

These values lead to some remarks :

- The acoustic mass ratio q is lower than 1. This is not surprising because the loudspeaker was measured in free-air where the total air load mass is that of a single face.
- More surprising is the value Q_l of leakage losses who is very high and can be considered as infinite.

- The identified value of the system compliance ratio α leads to compute the speaker V_{as}. Taking $\beta = 1$ because case A is for unlined enclosure, we get $V_{as} =$ 688 l.

  Identified values for case B to case E are :

q	f_{so}	Q_l	Q_a	Q_p
0.884	21.30	$+\infty$	29	38
h	α	$f_p(hz)$	β	$\beta V_b(l)$
1.787	2.488	38.06	1.012	276

Case B :

q	f_{so}	Q_l	Q_a	Q_p
0.866	21.08	$+\infty$	30	45
h	α	$f_p(hz)$	β	$\beta V_b(l)$
1.764	2.431	37.19	1.035	283

Case C :

q	f_{so}	Q_l	Q_a	Q_p
0.862	21.04	$+\infty$	30	36
h	α	$f_p(hz)$	β	$\beta V_b(l)$
1.619	2.415	34.09	1.042	285

Case D :

q	f_{so}	Q_l	Q_a	Q_p
0.859	21.00	$+\infty$	30	29
h	α	$f_p(hz)$	β	$\beta V_b(l)$
1.462	2.399	30.71	1.049	287

Case E :

Results are consistent with those expected :

- The absorption losses Q_a decrease when lining material is placed inside the enclosure, going from 63 to 29.
- β increases the more the enclosure is filled (the apparent volume goes from 273.5 liters up to 287 liters).
- The process of blocking one or two ports reduces significantly the resonant frequency f_p.

We can also remark that :

- The acoustic mass ratio q decreases as the enclosure is filled.
- Blocking the vents increase the β coefficient.

The last point is maybe due to the fact that the vents were blocked with acoustic foam.

The next step was to compute the free-field frequency response. The pressure inside the enclosure was measured with REW [7] and a calibrated Dayton EMM6 microphone.

The figure 11 shows the measurement with no smoothing for case A.

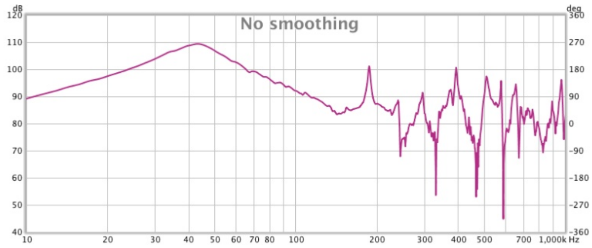

Figure 11: Pressure inside the enclosure

It can be seen that the measurement is smooth up to 150 hz. From that point, stationary waves inside the enclosure distort significantly the response.

The simulation of the response in case A is depicted figure 12. This figure superimposes the theoretical response calculated with the identified parameters and the measured response in 1/12 octaves transformed according to the equations of section 4.

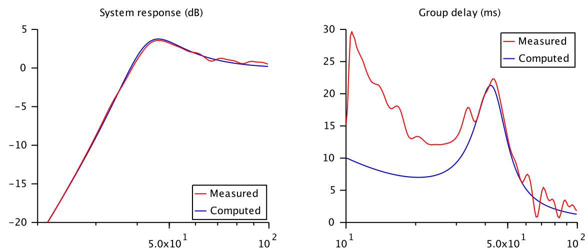

Figure 12: System response and group delay of case A

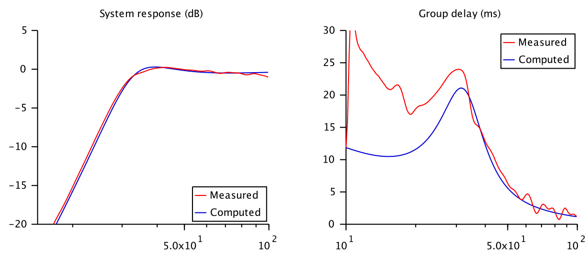

Figure 13: System response and group delay of case E

One can note the very good prediction of the theoretical model. The highest deviation between the theoretical and measured responses is less than 0.6 dB.

The measured group delay broadly follows the theoretical one. The deviation below 20 Hz is probably due to the cutoff frequency of the microphone which is 18 hz.

The response of the speaker in case A leads to a 35.2 hz cutoff frequency and a peak of 3.9 db at 46.2 hz.

Curves in figure 13 shows the case E. Blocking 2 vents on 6 has a very beneficial effect on the frequency response. The peak amplitude is no longer than 0.28 dB and the cutoff frequency goes down to 29.8 hz.

Once the parameters have been estimated, one can identify a new set of parameters using the script *<Measure Vented-Box 2.sce>* with the identified value of Q_{mo}. This leads to compute two new values of q with :

$$q_1 = (f_{so}/f_s)^2 \tag{26}$$

$$q_2 = (Q_{es}/Q_{eo})^2 \tag{27}$$

Case E leads to q_1=0.86 and q_2=0.82. These values are close to the initial value of q which is 0.86. This shows the consistency of the estimation.

The configuration E was retained to listen to these speakers.

Conclusion

This article presents Scilab scripts to identify the vented-box parameters and compute the free-field system frequency response from measurement of the pressure within the enclosure.

The following conclusions can be drawn from this study :

- It is not possible to simultaneously identify Q_{mo}, Q_l, Q_a, Q_p from impedance measurements. However when the Thiele and Small parameters of the loudspeaker are known, by introducing the mass ratio parameter q, it is possible to identify all Q losses factors of the enclosure as well as the system tuning ratio and the system compliance ratio.
- To increase accuracy of the identified parameters, one has to use both magnitude and phase of the impedance measurements.
- Enclosure leakage losses are accepted to be dominant in vented-box enclosures. This was not the case in this study where leakage losses were found to be neglectable.
- The simulation of the system frequency response with the identified parameters proved to be very close to the the free-field system response. Variance analysis shows that the accuracy of the simulated response is better than 0.6 db.
- The estimation of the vented-box parameters from the impédance measurements allows effective fine-tuning of a such loudspeaker system.

A The least square estimation method

This section summarizes some properties of the least square method. More details can be found in reference [9].

Let z a vector of m observations and x a vector of p variables. Let suppose that the measurement equation is linear, so that we get :

$$Jx = z + \epsilon \tag{28}$$

where J is a matrix of m rows and p columns. This model includes the hypothesis :

- The matrix J is of rank p
- The measurement noise has zero mean : $E(\epsilon) = 0$
- The measurement noise covariance matrix is : $V(\epsilon) = E(\epsilon\epsilon^\intercal) = \sigma^2\Gamma$

where Γ is a known positive definite square matrix of order m.

The least square estimation of x, noted $\hat{x}$ is given by :

$$\hat{x} = (J^\intercal\Gamma^{-1}J)^{-1}J^\intercal\Gamma^{-1}z \tag{29}$$

Let us note $A = J^\intercal\Gamma^{-1}J$. The variance of the estimator $\hat{x}$ is :

$$V(\hat{x}) = \sigma^2 A^{-1} \tag{30}$$

It can be demonstrate that the least square estimator is the minimum variance estimator within the class of unbiased linear estimator.

Let us suppose now that the exact values of $p-1$ variables of x are known. The estimation of the unknown component x_i is obtain by regression through the origin. We can write :

$$z' = J_i x_i + \epsilon \tag{31}$$

with :

$$z' = z - \sum_{j\neq i} J_j x_j \tag{32}$$

and where vector J_i is the i$^{\text{th}}$ column of matrix J. The variance of x_i is :

$$V_i = \frac{\sigma^2}{J_i^\intercal\Gamma^{-1}J_i} \tag{33}$$

Let us return to the normal situation where all variables of x is to be estimated. It can be demonstrate that the least square estimator satisfies :

$$\forall i \ V(\hat{x}_i) > \frac{\sigma^2}{J_i^\intercal\Gamma^{-1}J_i} \tag{34}$$

The ratio $V(\hat{x}_i)/V_i$ is called the variance inflation factor of the parameter i and is written VIF$_i$. This number is always greater than or equal to 1. The variance inflation factor of a parameter can be very large. It is related to the lack of observability of the parameter in question.

Let us perform a non-singular linear transformation in parameter space x so that :

$$\overline{x} = Kx \tag{35}$$

where K is a diagonal matrix with :

$$k_i = \sqrt{J_i^T \Gamma^{-1} J_i} \tag{36}$$

Equation (28) leads to :

$$JK^{-1}\overline{X} = z + \epsilon \tag{37}$$

Thus :

$$\overline{J}\,\overline{X} = z + \epsilon \tag{38}$$

with $\overline{J} = JK^{-1}$. The matrix $\overline{A}$ related to the least square estimation of $\overline{x}$ is written :

$$\overline{A} = \overline{J}^T \Gamma^{-1} \overline{J} = K^{-1} A K^{-1} \tag{39}$$

All of the diagonal elements of this matrix are equal to 1. It is said that this matrix has the form of a correlation matrix.

Diagnosing approximative collinearity is based on the spectral analysis of $\overline{A}$. Indeed, if s^i (for i between 1 and p) is an orthonormal basis composed of eigenvectors of $\overline{A}$, and the vector s_j is associated with the eigenvalue μ_j, then :

$$\mathrm{FIV}_i = \sum_j \frac{s_{ij}^2}{\mu_j} \tag{40}$$

where s_{ij} is the i$^{\text{th}}$ component of the vector s_j. This relation shows that small eigenvalues μ_j leads to large variance inflation factors.

Let us note μ_{min} the smallest eigenvalue of $\overline{A}$.

If $p\mu_{min}$ is less than 10^{-3}, the lack of observability is said to be severe. If its value is between 10^{-3} and 10^{-2}, the lack of observability is said to be strong.

The choice of the parameter $p\mu_{min}$ comes from the relation :

$$\max_i \mathrm{FIV}_i \geq \frac{1}{p\mu_{min}} \tag{41}$$

Thus for severe lack of observability, there are variance inflation factors greater than 1000.

The eigenvectors corresponding to the small eigenvalues give the coefficients of the linear combination of normalized parameters which are most likely to be poorly observed. On the other hand, the parameters that do not appear in these combinations are well observed.

B Links to the Scilab scripts

http://www.volucres.fr/AudioHighEnd/resources/SciAudioBoxEn/SciAudioBox.sci
http://www.volucres.fr/AudioHighEnd/resources/SciAudioBoxEn/Measure-Vented-Box-1.sce
http://www.volucres.fr/AudioHighEnd/resources/SciAudioBoxEn/Measure-Vented-Box-2.sce
http://www.volucres.fr/AudioHighEnd/resources/SciAudioBoxEn/Simulation-Vented-Box.sce

C Links to impedance and pressure measurements of case E

http://www.volucres.fr/AudioHighEnd/resources/SciAudioBoxEn/Vented-box-Impedance.txt
http://www.volucres.fr/AudioHighEnd/resources/SciAudioBoxEn/Vented-box-Impedance.lim
http://www.volucres.fr/AudioHighEnd/resources/SciAudioBoxEn/Internal-Pressure.txt
http://www.volucres.fr/AudioHighEnd/resources/SciAudioBoxEn/Internal-Pressure.mdat

References

[1] Scilab : open source software for numerical computation : http://www.scilab.org/en

[2] R. H. Small, Vented-Box Loudspeakers Systems. Part I-IV. Journal of the Audio Engineering Society.

[3] D. B. Keele, Low-Frequency Loudspeaker Assessment by Near-Field Sound Pressure Measurement" Journal of the Audio Engineering Society.

[4] R. H. Small, Simplified Loudspeaker Measurements at Low Frequencies. Journal of the Audio Engineering Society.

[5] WinISD freeware speaker designing software : http://www.linearteam.dk

[6] LIMP program for the loudspeaker impedance measurement and loudspeaker parameters estimation : http://www.artalabs.hr

[7] REW Room EQ Wizard Room Acoustics Software : http://www.roomeqwizard.com

[8] Jean Hiraga : Réalisation de l'enceinte grave Onken. L'audiophile, decembre 1977

[9] Paul Legendre : Parametric estimation : the least square method. Cépadues.

[10] Jean Fourcade : SciAudioBox, utilitaires Scilab pour le calcul et l'optimisation d'enceintes acoustiques. http://www.volucres.fr.